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A study on laminar film boiling of liquid along isothermal vertical plates in a pool with consideration of variable thermophysical properties

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Abstract—Rigorous theoretical models for pool film boiling of both subcooled and saturated liquids along an isothermal vertical plate, taking into consideration all matching conditions including variable thermophysical properties, are established. The governing partial equations for the film boiling and their boundary conditions are conveniently transformed into corresponding dimensionless ordinary differential equations, and are solved by a successively iterative procedure at different wall and ambient temperatures for saturated/subcooled water.

1. INTRODUCTION

BROMLEY [1] first treated laminar film-boiling heat transfer of saturated liquid around a horizontal cylinder in a pool in 1950. Later, some researches [2–7] were made to analyze pool film boiling on a vertical plate, but only a few analyses [5, 7] took temperature-dependence of the fluid's thermophysical properties into account. McFadden and Grosh [5] developed the analysis of saturated film boiling in a pool where the temperature-dependence of density and specific heat were taken into account. Nishikawa *et al.* [7] made an analysis of pool film boiling as a variable property problem on the basis of the two-phase boundary layer theory, but only the effect of variation of the vapor's thermophysical properties with temperature was examined in the range of lower degree of subcooling ($T_s - T_\infty = 0, 20, 40°C$).

However, the temperature difference between heating surface and bulk liquid may be very large, and the studies of the authors [8–10] and other scientists have shown that the thermophysical property variations of gas and liquid with temperature could have great influences on their free convection, and therefore affect the pool film boiling of liquid. The purpose of the present work is to advance a general theory for pool film boiling of both saturated and subcooled liquids along a vertical flat plate, and to investigate rigorously the characteristics of the velocity and temperature fields.

2. THEORETICAL MODELS

2.1. *Governing partial differential equations for the pool film boiling of subcooled liquid*

The analytical model and coordinating system used in the research is shown in Fig. 1. The heated plate with uniform temperature, T_w, is submerged vertically in a liquid pool whose temperature is lower than the saturated temperature. We assume that the heating

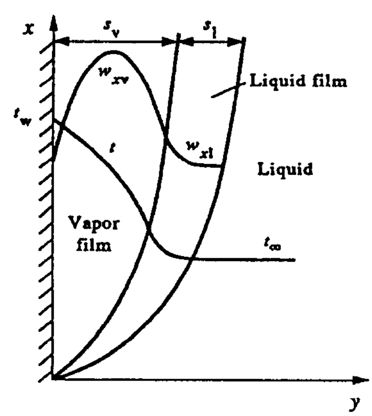

FIG. 1. Physical model and coordinate.

NOMENCLATURE

c_p specific heat at constant pressure [J kg^{-1} K^{-1}]

DT_l $(d\theta_l)/(d\eta_l)|_s$

DT_v $\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/4}\left[h_{fg}\mu_{v,s}(W_{xv,s}\eta_{v\delta}-4W_{yv,s})+\lambda_{v,s}(T_w-T_s)\frac{d\theta_v}{d\eta_v}\Big|_{\eta_v=\eta_{v\delta}}\right]\Big/\left[\lambda_{l,s}(T_s-T_\infty)\times\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{1/4}\left(\frac{\nu_{v,s}}{\nu_{l,\infty}}\right)^{1/2}\right]$

DW_{xl} $(dW_{xl})/(d\eta_l)|_s$

DW_{xv} $\frac{\mu_{v,s}}{\mu_{l,s}}\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{3/4}\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{-3/4}\times\left(\frac{\nu_{l,\infty}}{\nu_{v,s}}\right)^{1/2}\frac{dW_{xv}}{d\eta_v}\Big|_{\eta_v=\eta_v}$

g gravitational acceleration [m s^{-2}]

$Gr_{xv,s}$ local Grashof number for vapor film

$Gr_{xl,\infty}$ local Grashof number for liquid boundary layer

h_{fg} latent heat of vaporization [J kg^{-1}]

n_{cp} temperature exponent for specific heat of vapor

n_λ temperature exponent for thermal conductivity of vapor

n_μ temperature exponent for absolute viscosity of vapor

Pr Prandtl number, $\mu c_p/\lambda$

$Nu_{xv,w}$ local Nusselt number, $\alpha_x x/\lambda_{v,w}$

t temperature [°C]

T absolute temperature [K]

w velocity component [m s^{-1}]

W dimensionless velocity component

x, y coordinates.

Greek symbols

δ_v vapor film thickness [m]

η_v dimensionless coordinate variable for vapor film

η_l dimensionless coordinate variable for liquid film

θ_v dimensionless temperature for vapor film, $(T-T_s)/(T_w-T_s)$

θ_l dimensionless temperature for liquid boundary layer, $(T-T_\infty)/(T_s-T_\infty)$

λ thermal conductivity [W m^{-1} K^{-1}]

μ absolute viscosity [kg m^{-1} s^{-1}]

ν kinematic viscosity [m^2 s^{-1}]

ρ density [kg m^{-3}].

Subscripts

l liquid

s saturated state

v vapor

w at wall

∞ far from the wall surface.

plate surface is covered with continuous laminar vapor film, which moves upwards along the vertical plate and makes a layer of liquid near to the vapor film move upwards together with the vapor. Thus a two-phase boundary-layer is formed. Heat flux produced from the heating plate surface transfers through the two-phase boundary layer to the bulk liquid. Meanwhile, mass transfer is produced at the vapor–liquid interface due to the film boiling of liquid.

The governing equations of mass, momentum, and energy conservation for steady laminar free convection in the vapor and liquid boundary layers can be written as follow:

For vapor film,

$$\frac{\partial}{\partial x}(\rho_v w_{xv})+\frac{\partial}{\partial y}(\rho_v w_{yv})=0 \quad (1)$$

$$\rho_v\left(w_{xv}\frac{\partial w_{xv}}{\partial x}+w_{yv}\frac{\partial w_{xv}}{\partial y}\right)=g(\rho_{l,\infty}-\rho_v)+\frac{\partial}{\partial y}\left(\mu_v\frac{\partial w_{xv}}{\partial y}\right) \quad (2)$$

$$\rho_v c_{pv}\left(w_{xv}\frac{\partial t}{\partial x}+w_{yv}\frac{\partial t}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_v\frac{\partial t}{\partial y}\right). \quad (3)$$

For liquid film,

$$\frac{\partial}{\partial x}(\rho_l w_{xl})+\frac{\partial}{\partial y}(\rho_l w_{yl})=0 \quad (4)$$

$$\rho_l\left(w_{xl}\frac{\partial w_{xl}}{\partial x}+w_{yl}\frac{\partial w_{xl}}{\partial y}\right)=g(\rho_{l,\infty}-\rho_l)+\frac{\partial}{\partial y}\left(\mu_l\frac{\partial w_{xl}}{\partial y}\right) \quad (5)$$

$$\rho_l c_{pl}\left(w_{xl}\frac{\partial t}{\partial x}+w_{yl}\frac{\partial t}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_l\frac{\partial t}{\partial y}\right). \quad (6)$$

The boundary conditions will be

$y=0$,

$$w_{xv} = 0, \quad w_{yv} = 0, \quad t = t_w \tag{7}$$

$y = \delta_v$,

$$w_{xv,s} = w_{xl,s}; \tag{8}$$

$$\rho_{v,s}\left(w_{xv}\frac{\partial \delta_v}{\partial x} - w_{yv}\right)_s = \rho_{l,s}\left(w_{xl}\frac{\partial \delta_l}{\partial x} - w_{yl}\right)_s; \tag{9}$$

$$\mu_{v,s}\left(\frac{\partial w_{xv}}{\partial y}\right)_s = \mu_{l,s}\left(\frac{\partial w_{xl}}{\partial y}\right)_s; \tag{10}$$

$$-\lambda_{v,s}\left.\frac{\partial t_v}{\partial y}\right|_{y=\delta_v} = h_{fg}\rho_{v,s}\left(w_{xv}\frac{\partial \delta_v}{\partial x} - w_{yv}\right)_s - \lambda_{l,s}\left.\frac{\partial t_l}{\partial y}\right|_{y=\delta_v}; \tag{11}$$

$$t = t_s \tag{12}$$

$y \to \infty$,

$$w_{xl} \to 0, \quad \text{and } t \to t_\infty. \tag{13}$$

2.2. *Governing partial differential equations for pool film boiling of saturated liquid*

For the vapor film of a saturated liquid, the governing equations are thoroughly the same as equations (1)–(3). For liquid films in the saturated state of liquid, the temperature gradient is equal to zero, and hence, equation (6) drops, and the thermophysical properties of the liquid film are strictly constants, the governing partial differential equations can be simplified as follows:

$$\frac{\partial}{\partial x}(w_{xl}) + \frac{\partial}{\partial y}(w_{yl}) = 0 \tag{4'}$$

$$\left(w_{xl}\frac{\partial w_{xl}}{\partial x} + w_{yl}\frac{\partial w_{xl}}{\partial y}\right) = \nu_l\frac{\partial^2 w_{xl}}{\partial y^2}. \tag{5'}$$

The boundary conditions are the same as equations (7)–(13) except for $\lambda_{l,s}(\partial t_l/\partial y)_{y=\delta_v} = 0$ in equation (11).

2.3. *Similarity transformation of the governing partial equations and the boundary conditions*

In order to transform the above partial differential equations (1)–(6) and the boundary conditions (7)–(13) into a dimensionless system of equations, the following dimensionless variables are set up:

For vapor film,

$$\eta_v = \frac{y}{x}\frac{(Gr_{xv,s})^{1/4}}{\sqrt{2}} \tag{14}$$

$$Gr_{xv,s} = \frac{g(\rho_{l,\infty} - \rho_{v,w})x^3}{\nu_{v,s}^2\rho_{v,w}} \tag{15}$$

$$\theta_v = \frac{t - t_s}{t_w - t_s} \tag{16}$$

$$W_{xv} = \left[2\sqrt{(gx)}\left(\frac{\rho_{l,\infty} - \rho_{v,w}}{\rho_{v,w}}\right)^{1/2}\right]^{-1} w_{xv} \tag{17}$$

$$W_{yv} = \left[2\sqrt{(gx)}\left(\frac{\rho_{l,\infty} - \rho_{v,w}}{\rho_{v,w}}\right)^{1/2}(\tfrac{1}{4}Gr_{xv,s})^{-1/4}\right]^{-1} w_{yv}. \tag{18}$$

For liquid film in the case of subcooled liquid,

$$\eta_l = \frac{y}{x}\frac{(Gr_{xl,\infty})^{1/4}}{\sqrt{2}} \tag{19}$$

$$Gr_{xl,\infty} = \frac{g(\rho_{l,\infty} - \rho_{l,s})x^3}{\nu_{l,\infty}^2\rho_{l,s}} \tag{20}$$

$$\theta_l = \frac{t - t_\infty}{t_s - t_\infty} \tag{21}$$

$$W_{yl} = \left[2\sqrt{(gx)}\left(\frac{\rho_{l,\infty} - \rho_{l,s}}{\rho_{l,s}}\right)^{1/2}\right]^{-1} w_{x,l} \tag{22}$$

$$W_{xl} = \left[2\sqrt{(gx)}\left(\frac{\rho_{l,\infty} - \rho_{l,s}}{\rho_{l,s}}\right)^{1/2}(\tfrac{1}{4}Gr_{xl,\infty})^{-1/4}\right]^{-1} w_{yl}. \tag{23}$$

For liquid film in the case of saturated liquid,

$$\eta_l = \frac{y}{x}\frac{(Gr_{xl,\infty})^{1/4}}{\sqrt{2}} \tag{19'}$$

$$Gr_{xl,\infty} = \frac{gx^3}{\nu_{l,\infty}^2} \tag{20'}$$

$$W_{xl} = [2\sqrt{(gx)}]^{-1} w_{xl} \tag{22'}$$

$$W_{yl} = [2\sqrt{(gx)}(\tfrac{1}{4}Gr_{xl,\infty})^{-1/4}]^{-1} w_{yl}. \tag{23'}$$

With the assumed equations for dimensionless variables (14)–(23) the governing partial equations (1)–(6) are transformed into the following dimensionless system of equations:

For vapor film,

$$2W_{xv} - \eta_v\frac{dW_{xv}}{d\eta_v} + 4\frac{dW_{yv}}{d\eta_v} + \frac{1}{\rho_v}\frac{d\rho_v}{d\eta_v}(4W_{yv} - \eta_v W_{xv}) = 0 \tag{24}$$

$$\frac{\nu_{v,s}}{\nu_v}\left[W_{xv}\left(2W_{xv} - \eta_v\frac{dW_{xv}}{d\eta_v}\right) + 4W_{yv}\frac{dW_{xv}}{d\eta_v}\right] = \frac{\nu_{v,s}}{\nu_v}\frac{\rho_{v,w}}{\rho_v}\frac{\rho_{l,\infty} - \rho_v}{\rho_{l,\infty} - \rho_{v,w}} + \frac{d^2W_{xv}}{d\eta_v^2} + \frac{1}{\mu_v}\frac{d\mu_v}{d\eta_v}\frac{dW_{xv}}{d\eta_v} \tag{25}$$

$$Pr_v\frac{\nu_{v,s}}{\nu_v}(-\eta_v W_{xv} + 4W_{yv})\frac{d\theta_v}{d\eta_v} = \frac{d^2\theta_v}{d\eta_v^2} + \frac{1}{\lambda_v}\frac{d\lambda_v}{d\eta_v}\frac{d\theta_v}{d\eta_v}. \tag{26}$$

For liquid film in the case of subcooled liquid,

$$2W_{xl} - \eta_l\frac{dW_{xl}}{d\eta_l} + 4\frac{dW_{yl}}{d\eta_l} + \frac{1}{\rho_l}\frac{d\rho_l}{d\eta_l}(4W_{yl} - \eta_l W_{xl}) = 0 \tag{27}$$

$$\frac{\nu_{l,\infty}}{\nu_l}\left[W_{xl}\left(2W_{xl}-\eta_l\frac{dW_{xl}}{d\eta_l}\right)+4W_{yl}\frac{dW_{xl}}{d\eta_l}\right] = \frac{\nu_{l,\infty}}{\nu_l}\frac{\rho_{l,s}}{\rho_l}\frac{\rho_{l,\infty}-\rho_l}{\rho_{l,\infty}-\rho_{l,s}}+\frac{d^2W_{x,l}}{d\eta_l^2}+\frac{1}{\mu_l}\frac{d\mu_l}{d\eta_l}\frac{dW_{x,l}}{d\eta_l} \quad (28)$$

$$Pr_l\frac{\nu_{l,\infty}}{\nu_l}(-\eta_l W_{xl}+4W_{yl})\frac{d\theta_l}{\eta_l} = \frac{d^2\theta_l}{d\eta_l^2}+\frac{1}{\lambda_l}\frac{d\lambda_l}{d\eta_l}\frac{d\theta_l}{d\eta_l}. \quad (29)$$

For liquid film in the case of saturated liquid,

$$2W_{xl}-\eta_l\frac{dW_{xl}}{d\eta_l}+4\frac{4W_{yl}}{d\eta_l}=0 \quad (27')$$

$$\left[W_{xl}\left(2W_{xl}-\eta_l\frac{dW_{xl}}{d\eta_l}\right)+4W_{yl}\frac{dW_{xl}}{d\eta_l}\right]=\frac{d^2W_{xl}}{d\eta_l^2}. \quad (28')$$

With equations (14)–(23) the physical boundary conditions (7)–(13) for the subcooled liquid boiling are transformed equivalently to the following ones respectively:

$\eta_v = 0$,

$$W_{xv}=0,\quad W_{yv}=0,\quad \theta_v=0 \quad (30)$$

$\eta_v=\eta_{v\delta}$ and $\eta_l=0$,

$$W_{xl,s}=\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/2}\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{-1/2}W_{xv,s} \quad (31)$$

$$W_{yl,s}=-0.25\frac{\rho_{v,s}}{\rho_{l,s}}\left(\frac{\nu_{v,s}}{\nu_{l,\infty}}\right)^{1/2}\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/4}\times\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{-1/4}(W_{xv,s}\eta_{v\delta}-4W_{yv,s}) \quad (32)$$

$$\left.\frac{dW_{xl}}{d\eta_l}\right|_{\eta_l=0}=\frac{\mu_{v,s}}{\mu_{l,s}}\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{3/4}\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{-3/4}\times\left(\frac{\nu_{l,\infty}}{\nu_{v,s}}\right)^{1/2}\left.\frac{dW_{x,v}}{d\eta_v}\right|_{\eta_v=\eta_{v\delta}} \quad (33)$$

$$\left.\frac{d\theta_l}{d\eta_l}\right|_{\eta_l=0} = \frac{\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/4}\left[h_{fg}\mu_{v,s}(W_{xv,s}\eta_{v\delta}-4W_{yv,s})+\lambda_{v,s}(T_w-T_s)\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=\eta_{v\delta}}\right]}{\lambda_{l,s}(T_s-T_\infty)\left(\frac{\rho_{l,\infty}-\rho_{l,s}}{\rho_{l,s}}\right)^{1/4}\left(\frac{\nu_{v,s}}{\nu_{l,\infty}}\right)^{1/2}} \quad (34)$$

$$\theta_l=1 \quad (35)$$

$$\theta_v=0 \quad (36)$$

$\eta_l\to\infty$,

$$W_{xl}\to 0,\quad \theta_l=0. \quad (37)$$

With equations (14)–(18) or (19′)–(23′), the boundary conditions (7)–(13) for the saturated liquid film boiling are transformed to the following ones respectively:

$\eta_v = 0$,

$$W_{xv}=0,\quad W_{yv}=0,\quad \theta_v=1 \quad (30')$$

$\eta_v=\eta_{v\delta}$ and $\eta_l=0$, $W_{xl,s}=\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/2}W_{xv,s}$ (31′)

$$W_{yl,s}=-0.25\frac{\rho_{v,s}}{\rho_{l,s}}\left(\frac{\nu_{v,s}}{\nu_{l,\infty}}\right)^{1/2}\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{1/4}\times(W_{xv,s}\eta_{v\delta}-4W_{yv,s}) \quad (32')$$

$$\left.\frac{dW_{xl}}{d\eta_l}\right|_{\eta_l=0}=\left[\frac{\mu_{v,s}}{\mu_{l,s}}\left(\frac{\rho_{l,\infty}-\rho_{v,w}}{\rho_{v,w}}\right)^{3/4}\left(\frac{\nu_{l,\infty}}{\nu_{v,s}}\right)^{1/2}\left.\frac{dW_{xv}}{d\eta_v}\right|_{\eta_v=\eta_{v\delta}}\right] \quad (33')$$

$$h_{fg}\mu_{v,s}(W_{xv,s}\eta_{v\delta}-4W_{yv,s})+\lambda_{v,s}(T_w-T_s)\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=\eta_{v\delta}}=0 \quad (34')$$

$$\theta_v=0 \quad (35')$$

$\eta_l\to\infty$,

$$W_{xl}\to 0. \quad (37')$$

3. TREATMENT OF VARIABLE THERMOPHYSICAL PROPERTIES

For numerical calculation of the theoretical models, the temperature-dependence of the thermophysical properties of both vapor and liquid must be known. From our previous work [8], the temperature of vapor medium in saturation, T_s, can be taken as the reference temperature, and the thermophysical properties of the vapor can be expressed by simple power law as follows:

$$\frac{\mu_v}{\mu_{v,s}} = \left(\frac{T}{T_s}\right)^{n_\mu} \qquad (38)$$

$$\frac{\lambda_v}{\lambda_{v,s}} = \left(\frac{T}{T_s}\right)^{n_\lambda} \qquad (39)$$

$$\frac{\rho_v}{\rho_{v,s}} = \left(\frac{T}{T_s}\right)^{-1} \qquad (40)$$

$$\frac{\nu_v}{\nu_{v,s}} = \left(\frac{T}{T_s}\right)^{n_\mu+1} \qquad (41)$$

$$\frac{C_p}{C_{p_{v,s}}} = \left(\frac{T}{T_s}\right)^{n_{c_p}}. \qquad (42)$$

According to the typical experimental values in ref. [11], we obtain the following equations of water with the temperature range between 0 and 100°C [9]:

$$\rho_{wt} = -4.48\times10^{-3}t^2+999.9 \qquad (43)$$

$$\lambda_{wt} = -8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.56 \qquad (44)$$

and the absolute viscosity is quoted from ref. [12] as

$$\mu_{wt} = 10^{-3}\exp\left(-1.6-1150/T+(690/T)^2\right). \qquad (45)$$

By means of the definition of dimensionless variables η_v, θ_v, η_l and θ_l in equations (14), (16), (19), and (21), respectively, the corresponding thermophysical property factors of the theoretical models are expressed as follows:

For vapor film,

$$\frac{1}{\rho_v}\frac{d\rho_v}{d\eta_v} = -\frac{(T_w/T_s-1)\dfrac{d\theta_v}{d\eta_v}}{(T_w/T_s-1)\theta_v+1} \qquad (46)$$

$$\frac{1}{\mu_v}\frac{d\mu_v}{d\eta_v} = \frac{\mu_\mu(T_w/T_s-1)\dfrac{d\theta_v}{d\eta_v}}{(T_w/T_s-1)\theta_v+1} \qquad (47)$$

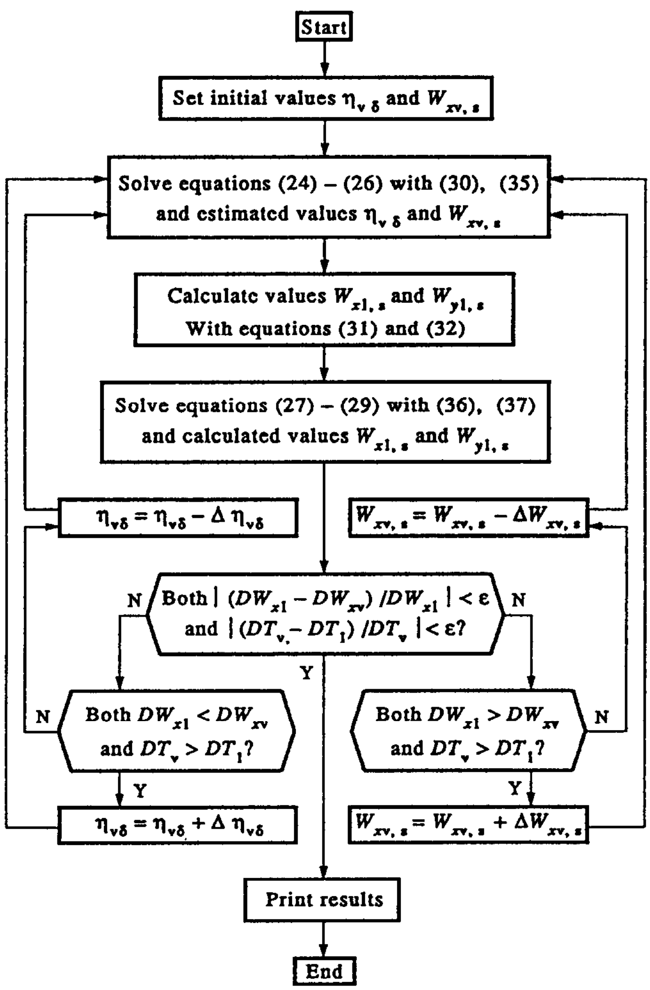

FIG. 2. The procedure chart of the calculation.

$$\frac{1}{\lambda_v}\frac{d\lambda_v}{d\eta_v} = \frac{n_\lambda(T_w/T_s-1)\frac{d\theta_v}{d\eta_v}}{(T_w/T_s-1)\theta_v+1} \quad (48)$$

$$\frac{\nu_{v,s}}{\nu_v} = [(T_w/T_s-1)\theta_v+1]^{-(n_\mu+1)}. \quad (49)$$

For liquid film in the case of subcooled liquid,

$$\frac{1}{\rho_l}\frac{d\rho_l}{d\eta_l} = \left[-2\times4.48\times10^{-3}t(t_s-t_\infty)\frac{d\theta_l}{d\eta_l}\right] \times(-4.48\times10^{-3}t^2+999.9)^{-1} \quad (50)$$

$$\frac{1}{\mu_l}\frac{d\mu_l}{d\eta_l} = \left(\frac{1150}{T^2}-2\frac{690^2}{T^3}\right)(t_s-t_\infty)\frac{d\theta_l}{d\eta_l} \quad (51)$$

$$\frac{1}{\lambda_l}\frac{d\lambda_l}{d\eta_l} = \left[(-2\times8.01\times10^{-6}t+1.94\times10^{-3})(t_s-t_\infty)\frac{d\theta_l}{d\eta_l}\right] \times(-8.01\times10^{-6}t^2+1.94\times10^{-3}t+0.562)^{-1} \quad (52)$$

where

$$t = (t_w-t_s)\theta_l+t_s. \quad (53)$$

4. NUMERICAL CALCULATION AND THE RESULTS

4.1. *Calculational procedure*

The procedure chart of the calculation is shown in Fig. 2. The values of $\eta_{v\delta}$ and W_{xvs} of the vapor film at the vapor–liquid interface are guessed first. These two values combined with equations (30) and (36) are used to solve the governing equations (24)–(26) in vapor film, so as to obtain W_{yvs}, $dW_{xv}/d\eta_v|_s$ and $d\theta_v/d\eta_v|_s$ at the interface. With $\eta_{v\delta}$, W_{xvs} and W_{yvs}, values of W_{xls} and W_{yls} can be calculated from equations (31) and (32). Then, W_{xls} and W_{yls} together with the boundary conditions (36) and (37) are used to solve the governing equations (27)–(29) to yield the values of $dW_{xl}/d\eta_l|_s$ and $d\theta_l/d\eta_l|_{\eta_l=0}$. Equations (33) and (34) are taken to adjudge convergence of the solutions for the two-phase boundary governing equations, and the calculation is successively iterated by changing the values of W_{xvs} and $\eta_{v\delta}$.

The calculative procedure for the saturated liquid pool film boiling is worked out in a similar way.

4.2. *Numerical results with discussions*

From ref. [8], the temperature parameters n_μ, n_λ and n_{c_p} of water vapor are 1.04, 1.185 and 0.003, respectively. Such low values of n_{c_p} makes it possible to actually treat c_p of vapor as a constant.

The numerical calculations have been carried out at wall temperatures of 277, 477, 577, 727 and 827°C, and ambient temperatures of 0, 10, 30, 50, 70, 90, and 100°C. The thermophysical properties needed in the calculations at the above specified temperature for both water and water vapor are taken from ref. [11] and summarized in Tables 1 and 2, respectively. Some numerical results of velocity and temperature profiles for the film boiling of subcooled saturated water are shown in Figs. 3–6.

Figures 3 and 4 show the velocity profile at ambient temperature 0, 70, 90 and 100°C, respectively. It is clear that, boundary layer thickness and velocity components of vapor film increase with increasing wall superheated degree, (t_w-t_s), or with decreasing subcooled degree of bulk water, (t_s-t_∞). The film thickness and the velocity components in vapor film increase more in the range of the low subcooled degree (t_s-t_∞) than in the range of the high subcooled degree (t_s-t_∞).

The temperature profiles at the same ambient temperature as the velocity profiles are shown in Figs. 5 and 6, respectively. It is obviously known that for a given subcooled degree the temperature gradient $d\theta_v/d\eta_v|_{\eta_v=0}$ decreases with the increase in the wall superheated degree. It is necessary to emphasize that the temperature profiles in the vapor film in this work are quite different from those of other solutions with constant thermophysical properties where the temperature profile in the vapor film has the results $d^2\theta_v/d\eta_v^2 > 0$ [2, 6].

By comparison with the pool film boiling of subcooled water, the vapor film thickness and velocity component in the saturated water pool film boiling increase greatly, because the heat transferred from the flat plate is all used for the vaporization of the saturated water.

The effects of the superheated degree on the surface and subcooled degree of the liquid on the velocity and temperature fields show that variable thermophysical properties of both vapor and liquid film media have great influence on the film boiling.

5. HEAT AND MASS TRANSFER

The heat transfer rate per unit area from the plate to the vapor film for the pool laminar film boiling of liquid can be expressed by Fourier's law. Combining equations (14) and (16), q_x is expressed as

$$q_x = -\lambda_{v,w}(t_w-t_s)\left(\tfrac{1}{4}Gr_{xv,s}\right)^{1/4}x^{-1}\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=0}. \quad (54)$$

The local heat transfer coefficient on the surface will be

$$\alpha_x = -\lambda_{v,w}\left(\tfrac{1}{4}Gr_{xv,s}\right)^{1/4}x^{-1}\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=0}. \quad (55)$$

The local Nusselt number, defined as $Nu_{xv,w} = \alpha_x x/\lambda_{v,w}$, could be

$$N_{xv,w} = -\left(\tfrac{1}{4}Gr_{xv,s}\right)^{1/4}\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=0}. \quad (56)$$

This shows that the local heat transfer coefficient is in direct proportion to dimensionless temperature gradi-

Table 1. Density of water vapor at atmospherical pressure at different wall temperature

t_w (°C)	277	377	477	577	727	827
$\rho_{v,w}$ (kg m^{-3})	0.338	0.2931	0.2579	0.2312	0.1996	0.1830

Table 2. Thermophysical properties values of water at different temperature

t_w (°C)	0	10.0	30.0	50.0	70.0	90.0	100.0
$\rho_{l\infty}$ (kg m^{-3})	999.8	999.8	995.8	988.1	977.7	965.1	958.4
$\nu_{l\infty}$ (10^{-6} m^2 s^{-1})	1.792	1.308	0.798	0.554	0.414	0.326	0.296
$\lambda_{l\infty}$ (10^{-3} W K^{-1} m^{-1})	562.0	582.0	615.1	640.5	659.5	672.8	677.3
$Pr_{l\infty}$	13.44	9.42	5.42	3.57	2.57	1.97	1.76

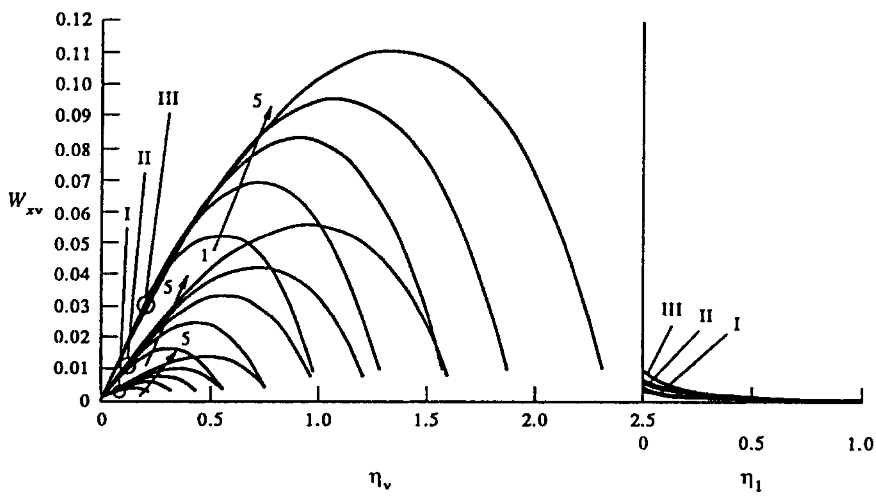

FIG. 3. Velocity profiles for laminar film boiling of subcooled water. I–III: t_∞ = 0, 70, and 90°C, respectively; 1–5: t_w = 277, 377, 477, 577 and 727°C, respectively.

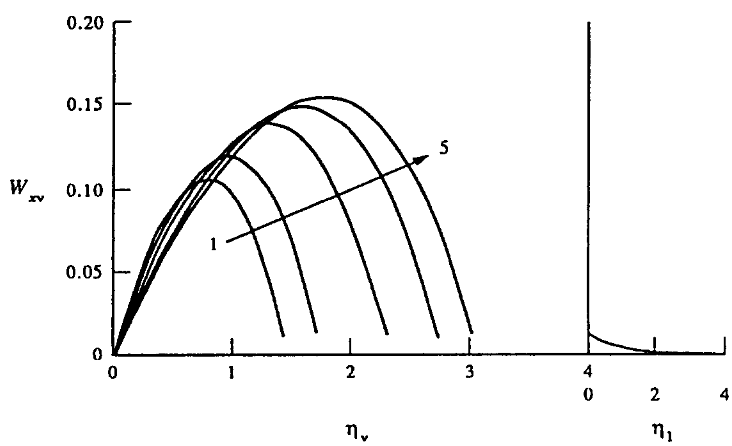

FIG. 4. Velocity profiles for laminar film boiling of saturated water. 1–5: t_w = 277, 377, 577, 727 and 827°C, respectively.

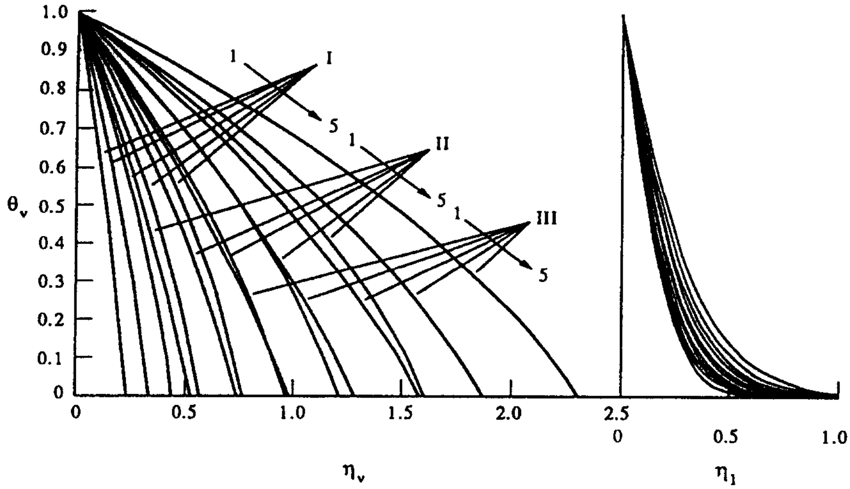

FIG. 5. Temperature profiles for laminar film boiling of subcooled water. I–III: $t_\infty = 0$, 70, and 90°C, respectively; 1–5: $t_w = 277$, 377, 477, 577 and 727°C, respectively.

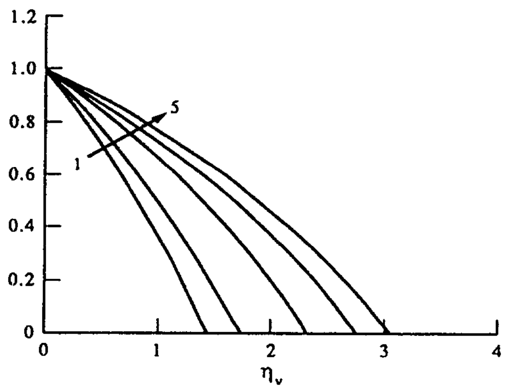

FIG. 6. Temperature profiles for laminar film boiling of saturated water. 1–5: $t_w = 277$, 377, 577, 727 and 827°C, respectively.

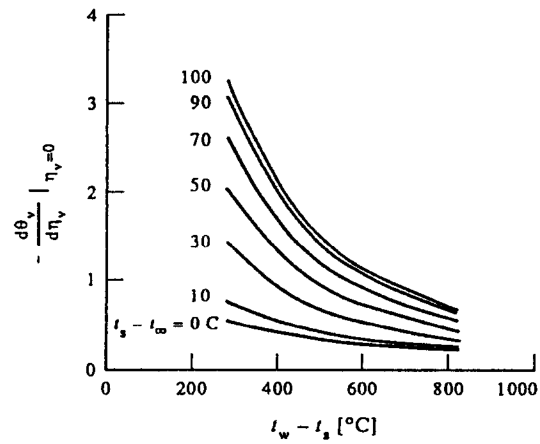

FIG. 7. Relation between $-(d\theta_v)/(d\eta_v)|_{\eta_v=0}$ and $(t_w - t_s)$.

ent $-d\theta_v/d\eta_v|_{\eta_v=0}$ on the surface. The numerical solutions for $-d\theta_v/d\eta_v|_{\eta_v=0}$ in both saturated and subcooled conditions of water are described in Table 3 and plotted as Fig. 7. We get the following correlation:

$$-\left.\frac{d\theta_v}{d\eta_v}\right|_{\eta_v=0} = \frac{\exp\left[A + B\times 10^{-2}(t_w - t_s) + C\times 10^{-4}(t_w - t_s)^2\right]}{t_w - t_s}. \quad (57)$$

For $0 \leqslant t_s - t_\infty \leqslant 30$°C,

$$A = 4.7356 + 7.407\times 10^{-2}(t_s - t_\infty) - 7.4\times 10^{-4}(t_s - t_\infty)^2$$

$$B = 0.1228 - 1.633\times 10^{-2}(t_s - t_\infty) - 2.71\times 10^{-4}(t_s - t_\infty)^2$$

$$C = 0.0086 + 1.092\times 10^{-3}(t_s - t_\infty) - 2.132\times 10^{-5}(t_s - t_\infty)^2. \quad (58)$$

For $30 \leqslant t_s - t_\infty \leqslant 100$°C,

$$A = 5.515 + 3.01\times 10^{-2}(t_s - t_\infty) - 1.4\times 10^{-4}(t_s - t_\infty)^2$$

$$B = -0.1073 - 6.6\times 10^{-4}(t_s - t_\infty) + 4.437\times 10^{-6}(t_s - t_\infty)^2$$

$$C = 0.0069 - 7.812\times 10^{-5}(t_s - t_\infty) - 4.82\times 10^{-7}(t_s - t_\infty)^2. \quad (59)$$

The calculated results by the correlation, equation (57), compare very well with the corresponding rigorous numerical solutions.

Table 3. Calculated results of $-d\theta_v/d\eta_v|_{\eta_v=0}$: (1) numerical solutions; (2) from equation (57) with equations (58) and (59)

$t_w - t_s$ (°C)		$t_s - t_w$ (°C) 0	10	30	50	70	90	100
277	(1)	0.5411	0.7733	1.4414	2.0672	2.6202	3.0810	3.2686
	(2)	0.5410	0.7731	1.4410	2.0550	2.6540	3.1040	3.2344
377	(1)	0.4285	0.5621	0.9684	1.3734	1.7328	2.0381	2.1609
	(2)	0.4250	0.5612	0.9671	1.3639	1.7523	2.0512	2.1431
477	(1)	0.3531	0.4387	0.7054	0.9872	1.2416	1.4577	1.5472
	(2)	0.3530	0.4383	0.7054	0.9819	1.2537	1.4690	1.5395
577	(1)	0.2997	0.3587	0.5438	0.7496	0.9384	1.0999	1.1686
	(2)	0.3012	0.3586	0.5434	0.7456	0.9457	1.1088	1.1657
727	(1)	0.2433	0.2802	0.3954	0.5317	0.6607	0.7720	0.8211
	(2)	0.2430	0.2800	0.3953	0.5293	0.6639	0.7785	0.8231
827	(1)	0.2158	0.2439	0.3313	0.4379	0.5412	0.6315	0.6688
	(2)	0.2112	0.2435	0.3320	0.4364	0.5423	0.6367	0.6760

If G_x is the mass flow rate of the liquid boiling through an area $1 \times X$ with unit width started from the bottom of the plate on the vapor–liquid interface, the mass flow rate through a unit area with unit width on the interface at the height x will be $dG_x/dx = \rho_{v,s}(W_{xv,s}(d\delta_v/dx) - W_{yv,s})_s$, i.e.

$$\frac{dG_x}{dx} = \rho_{v,s}\left[2\sqrt{(gx)}\left(\frac{\rho_{l,\infty}}{\rho_{v,w}}-1\right)^{1/2} W_{xv,s}\left.\frac{d\delta_v}{dx}\right|_s - 2\sqrt{(gx)}\left(\frac{\rho_{l,\infty}}{\rho_{v,w}}-1\right)^{1/2}(\tfrac{1}{4}Gr_{xv,s})^{-1/4}W_{yv,s}\right] \quad (60)$$

where

$$\delta_v = \eta_{v\delta}\left[\frac{1}{4}\frac{g}{\nu_{v,s}^2}\left(\frac{\rho_{l,\infty}}{\rho_{v,w}}-1\right)\right]^{-1/4} x^{1/4} \quad (61)$$

and

$$\left.\frac{d\delta_v}{dx}\right|_s = \eta_{v\delta}\left[\frac{1}{4}\frac{g}{\nu_{v,s}^2}\left(\frac{\rho_{l,\infty}}{\rho_{v,w}}-1\right)\right]^{-1/4}\tfrac{1}{4}x^{-3/4}$$

$$\left.\frac{d\delta_v}{dx}\right|_s = \tfrac{1}{4}\eta_{v\delta}(\tfrac{1}{4}Gr_{xv,s})^{-1/4}. \quad (62)$$

Hence equation (60) can be simplified as

$$\frac{dG_x}{dx} = \mu_{v,s}(\tfrac{1}{4}Gr_{xv,s})^{1/4}(\eta_{v\delta}W_{xv,s} - 4W_{yv,s})x^{-1}. \quad (63)$$

The mass flow rate G_x is the integral as follows:

$$G_x = \int_0^x \frac{dG_x}{dx}\,dx.$$

Hence,

$$G_x = \tfrac{4}{3}\mu_{v,s}(\tfrac{1}{4}Gr_{xv,s})^{1/4}(\eta_{v\delta}W_{xv,s} - 4W_{yv,s}) \quad (64)$$

or in dimensionless form,

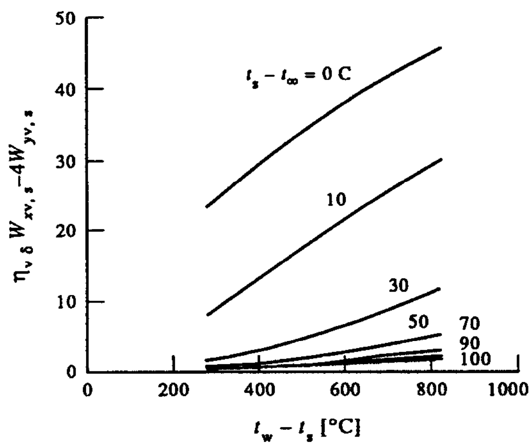

FIG. 8. Relation between $(\eta_{v\delta}W_{xv,s} - 4W_{yv,s})$ and $(t_w - t_s)$.

$$\frac{G_x}{\mu_{v,s}} = \tfrac{4}{3}(\tfrac{1}{4}Gr_{xv,s})^{1/4}(\eta_{v\delta}W_{xv,s} - 4W_{yv,s}). \quad (65)$$

The corresponding values are also plotted in Fig. 8.

6. CONCLUSION

From the analysis and calculated results, the following points can be concluded.

1. The dimensionless velocity components W_x and W_y, have been put forward in this work. As W_x and W_y have definite physical meanings, the corresponding solutions of the models can be understood easily.

2. For the pool laminar film boiling of subcooled and saturated liquid, the vapor film thickness and velocity components increase with increasing superheated degree $(t_w - t_s)$ or with decreasing subcooled degree $(t_s - t_\infty)$ of bulk liquid. The increase of the velocity components and the thickness of the vapor film become remarkably large when T_∞ reaches the saturated temperature.

3. The temperature gradient on the plate is steeper with higher subcooled degree and/or with lower superheated degree ($t_w - t_s$) of the surface than that with higher one.

4. The correlation, equation (57), will be useful for prediction of wall heat transfer, $Nu_{xv,w}$, while equation (65) is valuable to predict the mass transfer rate. Of course, such prediction does not include the radiative heat transfer across the vapor film, which needs to be further investigated.

REFERENCES

1. L. Bromley, Heat transfer in stable film boiling, *Chem. Engng Prog.* **46**(5), 221–227 (1950).
2. J. C. Y. Koh, Analysis of film boiling on vertical surface, *J. Heat Transfer* **84C**(1), 55–62 (1962).
3. R. D. Cess, Analysis of laminar film boiling from a vertical flat plate, Research Report 405 FF 340-R2-X, Westinghouse Research Lab., Pittsburgh, PA, March (1959).
4. E. M. Sparrow and R. D. Cess, The effect of subcooled liquid on laminar film boiling, *J. Heat Transfer* **84C**(2), 149–156 (1962).
5. P. W. McFadden and R. J. Grosh, An analysis of laminar film boiling with variable properties, *Int. J. Heat Mass Transfer* **1**, 325–335 (1961).
6. K. Nishikawa and T. Ito, Two-phase boundary-layer treatment of free convection film boiling, *Int. J. Heat Mass Transfer* **9**, 103–115 (1966).
7. K. Nishikawa, T. Ito and K. Matsumoto, Investigation of variable thermophysical property problem concerning pool film boiling from vertical plate with prescribed uniform temperature, *Int. J. Heat Mass Transfer* **19**, 1173–1182 (1976).
8. De-Yi Shang and Bu-Xuan Wang, Effect of variable thermophysical properties on laminar free convection of gas, *Int. J. Heat Mass Transfer* **33**, 1387–1395 (1990).
9. D. Y. Shang, B. X. Wang, Y. Wang and Y. Quan, Study on liquid laminar free convection with consideration of variable thermophysical properties, *Int. J. Heat Mass Transfer* **36**, 3411–3419 (1993).
10. D. Y. Shang and B. X. Wang, Effect of variable thermophysical properties on laminar free convection of polyatomic gas, *Int. J. Heat Mass Transfer* **34**, 746–756 (1991).
11. *VDI-Waermeatlas*, Berechnungs Blaetter fuer den Waermeuebertragung, Vol. 5, Erweiterte Auflage. VDI Verlage GmbH, Duesseldorf (1988).
12. J. Q. Chang *et al.*, *Real Fluid Mechanics* (in Chinese). Tsinghua University Press, Beijing, China (1986).